Math 1910, Section 9, Test #4 (Chapter 4) Name______________________

Each of these 10 problems has a value of 5 points.

1. Find all the critical numbers of each of the following functions:

(a) $f(x) = x^4(x-3)^3$

Use the product rule to calculate f'(x). The critical numbers are 0, 3 and 12/7

(b) $f(x) = 2 \cos \theta + \sin^2 \theta$

f'(x)=0 whenever $\cos \theta=1$ or $\sin \theta=0$. The critical numbers are all integer multiples of π

2. Find the absolute maximum value of the function $f(x) = x/(x^2+25)$ on the interval [-2,4]. Note that $f(-2)=-2/29\approx-0.069$ and $f(4)=4/41\approx0.098$.

The only critical number for f(x) are +5 and -5, which do not lie in the interval of interest. Thus, the absolute maximum must be 0.098, which occurs at the endpoint x=4.

3. (a) What is the average slope of the function $f(x)=x^2$ on the interval [0,4]?

The slope of the secant line from (0,0) to (4,16) is 4.

(b) Find a point on the graph where the slope is equal to that average slope.

f'(x) = 2x = 4 implies that x=2. Thus, the slope has the average value at the point (2,4).

4. Use Rolle's theorem to prove that the polynomial $p(x) = x^5+4x^3+ 4x - 5$ has exactly one real root. [Hint: Use *proof by contradition* to make your argument … i.e., suppose there are two or more real roots and show that forces a false statement to be true]

(a) Explain why we know that p(x) has a (one or more) real root?

p(0) = -5 and p(1) = +4. The intermediate value theorem tells us that p(c) must be zero for some number c between 0 and 1.

(b) Explain why we know there can't be more than one real root?

If p(x) had two real roots then, according to Rolle's theorem, p'(x) would have a root between those roots of p(x). It is easy to see that p'(x) is greater than zero for all real numbers, so it is impossible for p(x) to have two real roots

5. For the function $f(x) = x^3 – 6x^2 +9x +1$

(a) find all the intervals where the graph of this function is increasing

f'(x)=3(x-3)(x-1) which is positive when x>3 and when x<1.

(b) find all the intervals where the graph of this function is concave downward

f''(x)=6(x-2) which is negative when x<2

6. Use L'Hospital's rule to compute each of the following limits:

(a) $$\lim_{x\to\infty}\frac{x^3}{3^x}=\lim_{x\to\infty}\frac{3x^2}{3^x\cdot\ln(3)}=\lim_{x\to\infty}\frac{6x}{3^x\cdot\ln(3)^2}\lim_{x\to\infty}\frac{6}{3^x\cdot\ln(3)^3}=0$$

(b) $$\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)}=\lim_{x\to 0}\frac{2\cos(2x)}{3\cos(3x)}=2/3$$

7. Use L'Hospital's rule to compute each of the following, after rewriting the expressions in an appropriate form

(a) $$\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)=\lim_{x\to 1^+}\left(\frac{x\cdot\ln(x)-x+1}{(x-1)\cdot\ln(x)}\right)=\lim_{x\to 1^+}\left(\frac{\ln(x)}{1+\ln(x)-1/x}\right)=\lim_{x\to 1^+}\left(\frac{1/x}{1/x+1/x^2}\right)=1/2$$

(b) $$\lim_{x\to 0^+}\sqrt{x}\cdot\ln(3x)=\lim_{x\to 0^+}\frac{\ln(3x)}{x^{-1/2}}=\lim_{x\to 0^+}\frac{1/x}{(-1/2)x^{-3/2}}=\lim_{x\to 0^+}-2x^{1/2}=0$$

8. What is the smallest vertical distance between the graphs of $y=x^2+1$ and $y=x-x^2$?

The vertical distance is $D(x)=(x^2+1)-(x-x^2) = 2x^2-x+1$
$D'(x) = 4x-1$ which is zero when $x=1/4$. This gives the minimum distance of 7/8 units.

9. Find a particular antiderivative and the most general antiderivative for each of the following:

f(x)	**particular antiderivative**	**most general antiderivative**
$18x^2 - 14x + 9$	$6x^3 - 7x^2 + 9x$	$6x^3 - 7x^2 + 9x + C$
$8x^{1/7} - 10x^{1/9}$	$7x^{8/7} - 9x^{10/9}$	$7x^{8/7} - 9x^{10/9} + C$
$3\cos(x) - 6\sin(x)$	$3\sin(x) + 6\cos(x)$	$3\sin(x) + 6\cos(x) + C$

10. Find the equation of motion of a rock tossed into the air with an initial upward velocity of 12 ft/sec from the top of Fall Creek Falls (a 256 foot tall waterfall). Assume that the only force acting of the object is the force of gravity which is a constant 32 ft/sec^2 in the downward direction. Use the bottom of the waterfall as the position with value zero, so that the initial position is s(0)=256.

acceleration = $s''(t) = -32$

velocity = $s'(t) = -32t + 12$ because the initial velocity is +12

position = $s(t) = -16t^2 + 12t + 256$ because the initial position is +256